Self-Calibration in Radio Astronomy

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Insufficiency of Standard Calibration

$$V'_{\mathrm{ij}}(t,u,v)=g_i(t).g_j^*(t).V_{\mathrm{ij}}(u,v)$$

$g(t)$ varies with time during Target source observations.

Main problem from Ionospheric phase variation.
Causes point source to smear out and low level fake structures in a map.

Gain error:
$\sigma_G^2=\frac{\sigma_V^2}{(N-3)S^2}$ (N is no. of antennas).

Self-calibration

N complex gain errors corrupt $\left(\frac{N(N-1)}{2}\right)$ visibilities.

Allowing N Gains to be variable, provide $\left(\frac{N(N-1)}{2}\right)-N$ constraints.

Self-calibration involves minimising

$$R = \sum_k \sum_{i,j,i \neq j} |V_{\mathrm{ij}}(t_k) - g_i(t_k) g_j^*(t_k) . V_{\mathrm{ij}}^M(t_k)|^2$$

or

$$R = \sum_k \sum_{i,j,i \neq j} |V_{\mathrm{ij}}^M(t_k)|^2 |X_{\mathrm{ij}}(t_k) - g_i(t_k) g_j^*(t_k)|^2 \longrightarrow (1)$$

where $X_{\mathrm{ij}}(t_k) = \frac{V_{\mathrm{ij}}(t_k)}{V_{\mathrm{ij}}^M(t_k)}$.

Steps:

1. Make an initial model of the source using preliminary Cal. and acceptable source structures.

2. Solve for complex gains with certain averaging time (T) using Eq.(1).

3. Find the corrected visibility $V_{\mathrm{ij,corr}}(t) = \frac{V_{\mathrm{ij}}(t)}{g_i(t) g_j^*(t)}$

5. Form a new model from the image made from the corrected data.

6. Go to 2, unless satisfied.

How does it work

Consider
$I(l,m)=\frac{1}{M}\sum_{k=1}^{M} V(u_k,v_k)e^{2\pi i(u_k l+v_k m)}$ in 1-D.

$I(l)=\frac{1}{M}\sum_{k=1}^{M} V(u_k)e^{2\pi i(u_k l)}$

Assume $N-1$ antennas to have 0 phases.
1 to have **1 Rad** as phase.

Real source of unit strength at $l=0$.
Recall

$V'_{\rm ij}(t,u,v)=g_i(t).g_j^*(t).V_{\rm ij}(u,v)$

$I(l)=\frac{1}{M}\Big(\sum_{k=1}^{M-(N-1)} e^{2\pi i(u_k l)}+\sum_{M-N}^{M} e^{i}.e^{2\pi i(u_k l)}\Big)$
or, $I(0)=\frac{1}{M}\big((M-N+1)+\sum_{M-N+1}^{M} e^{i}\big)$
or, $I(0)=\left(\left(1-\frac{(N-1)\times 2}{N^2}\right)+\left(\frac{(N-1)\times 2}{N^2}\right)\cos(57^{\circ})\right)$
During Self-cal, one would divide
$\rm V_{ij}(u,v)$ by FT(I).

As one can see, the left side being real, only the real part of the right side of the equation need to be computed. When $N\gg 1$, $I(0)\sim 1-\frac{1}{N}$.

Note that the peak emission from the baselines with antenna $j = N$ would be at $l = -\frac{1}{2\pi . u_k}$. As you can see that it is off-centre due to phase error. However, as u_k for different baselines differ, it does not form a single source, but an emission that appear to be smeared out. Most often its peak is weak and do not form part of the model source/sources.

During self-cal, the prominent source at $l = 0$ will be used as source model. Then, $(X_{\text{ij}})_{\text{amp}} = \frac{1}{\left(1 - \frac{1}{N}\right)}$.
As mentioned, the amplitude part of the gains are not modified in the first few iterations of self-cal, when the antenna based phases are corrected.
The phase part of $X_{\text{ij}} = e^{(\theta_{\text{ij}} - \theta_{\text{ij}}(\text{Model}))}$.
For all baselines except involving the antenna N that had a phase error of 1 rad, will be zero.
However, phase part of $X_{\text{iN}} = e^{(1-0)}$.
During factorising the baseline based gains to antenna based ones, the phase of $g_N = 1$ rad, and as all baselines are corrected by $V_{\text{ij,corr}}(t) = \frac{V_{\text{ij}}(t)}{g_i(t) g_j^*(t)}$, the data for all V_{iN} get corrected.