Polarisation 5

- Absorption / loss on the Poincare sphere (PS) Motivated by optics of absorbing crystals
- Connection with Lorentz transformations
- Polarisation and phase combined in three dimensions
- Linear polarisation combined with direction of propagation

Losslessness

- Preserves intensity , so is a unitary matrix, satisfying $\boldsymbol{U\,U^{+}=I_2^{+}}$ acting on the two electric field amplitudes. Its determinant is of the form *exp (i α)*
- Stokes parameters (SP) transform as $\boldsymbol{S'=U\,S\,U^{+}}$
- ***U*** has 4 real parameters because of normalisation and orthogonality of the rows
- Going to the PS, we are not interested in the phase we can confine ourselves to those with unit determinant
- These have three real parameters
- Since we preserve both *I* (trace) and I^2-Q^2-U^2 -V^2 , hence rigid rotations in the QUV space, or on the PS

Loss

- A general linear device has 8 real parameters.
- The intensity is multiplied by *det* $(G\,\mathbf{G}^{+})$ we can take ***det G =1***, we are left with 6 parameters.
- Our lossless devices only account fo 3, so we need three more, and they cannot be rigid rotations.
- Acting on *I,Q,U,V* they are Lorentz transformations

Lorentz transformations

- We dont want to revisit rigid rotations
- Keep V axis fixed, and do not allow Q and U to rotate about it.
- The only possibility is *(1,z) -> (1, rz)* where r is real
- *cot(θ/2)* is multiplied by *r*
- Becomes movement along longitudes by an amount which vanishes at the poles – “circular dichroism”
- The two circular states are still ‘eigenstates’, i.e are unaltered in polarisation (though not in intensity)

What happens when we superpose a rigid rotation?

- If the rotation is along the V axis, then we have a kind of spiralling motion on the PS
- If the rotation is along the Q (or U) axis, then we get a shift of the two opposite directions which are left invariant, to two non opposite directions
- An absorbing crystal can show interference fringes with unpolarised incident light and without an analyser – they are week, but attracted the attention of Raman and Pancharatnam to this family of crystals

What happens if they merge?

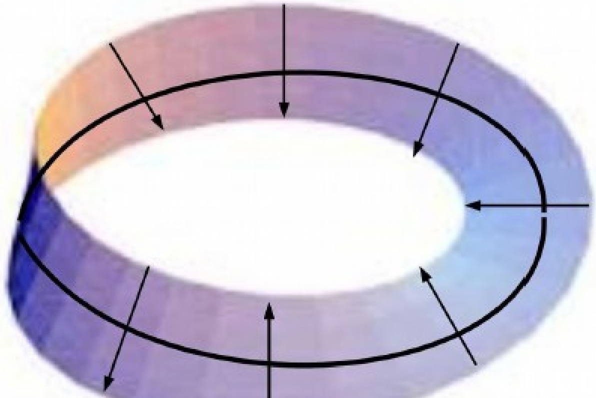
- All states, incuding the orthogonal state, now have to flow towards this one state
- But the paths which take on the PS are such that no other point is fixed!
- Near the pole, they look like dipole field lines
- Algebraically, they represent a case where the two eigenvectors collapse into 1 – a ‘Jordan form’

Depolarisation

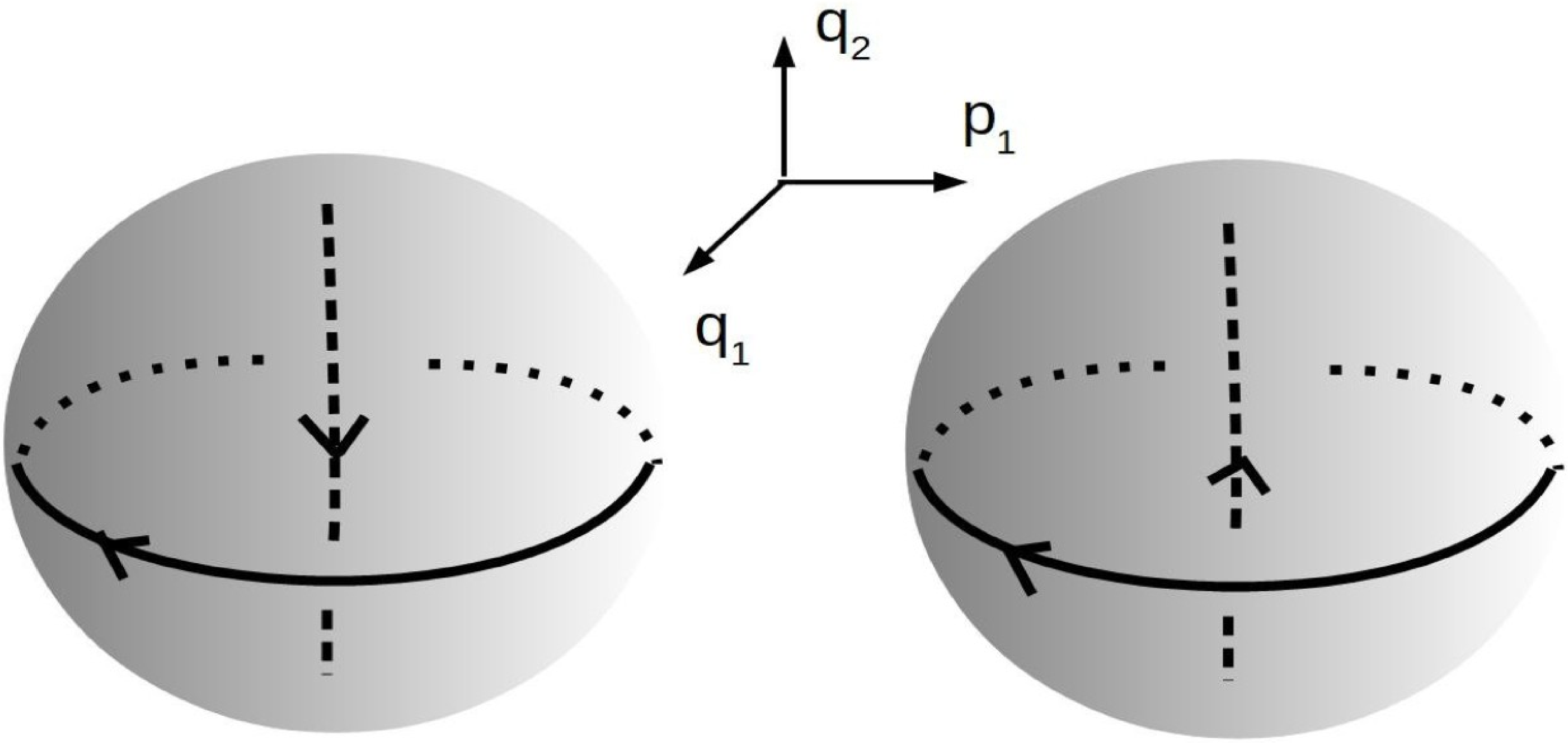
- Can occur because of a ‘Faraday screen’ . Westerbork work on galactic background
- Light passing through a polycrystalline aggregate
- Cannot be repesented by 2x2 matrices unless one averages over them as well
- Needs 4x4 (real) “Mueller matrices” acting on the Stokes parameters

Beyond Cartesian duality

- No difficulty in assigning x and y to points on a plane, or an angle θ and a height z to a point on a cylinder
- Not so easy to assign an angle θ and a height z to points on a Mobius band. We need two 'patches' to do the job
- This is only locally a 'Cartesian product, but globally 'twisted'
- This kind of situation led a few mathematicians to define ? invent ? discover? fibre bundles, around (1930-1940) . The geometric object that captures how polarisation and phase are intertwined, is now known as the 'Hopf fibration' after Heinz Hopf (1931)

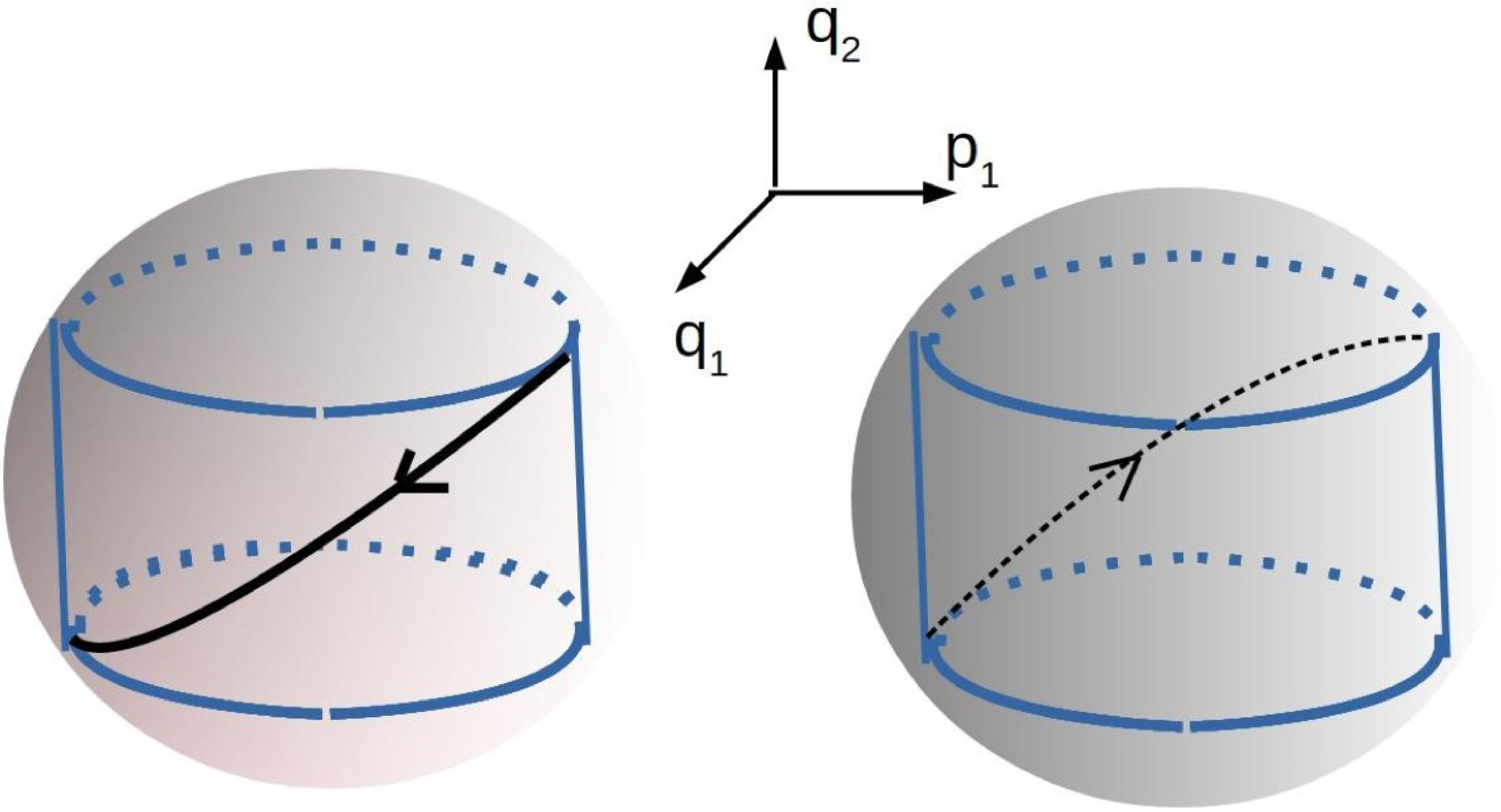


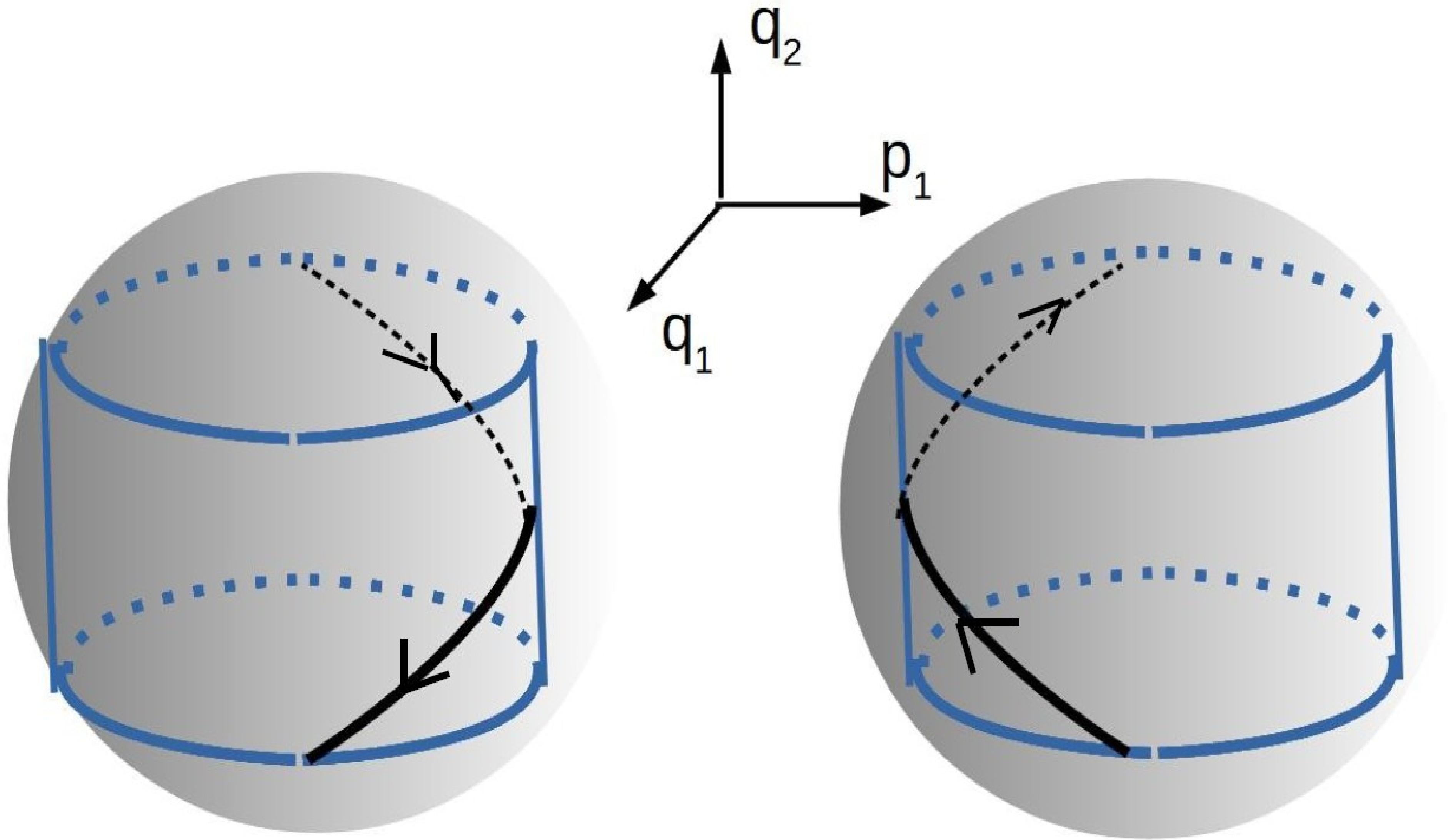
Not all fibre bundles are twisted! The cylinder and the plane are examples.



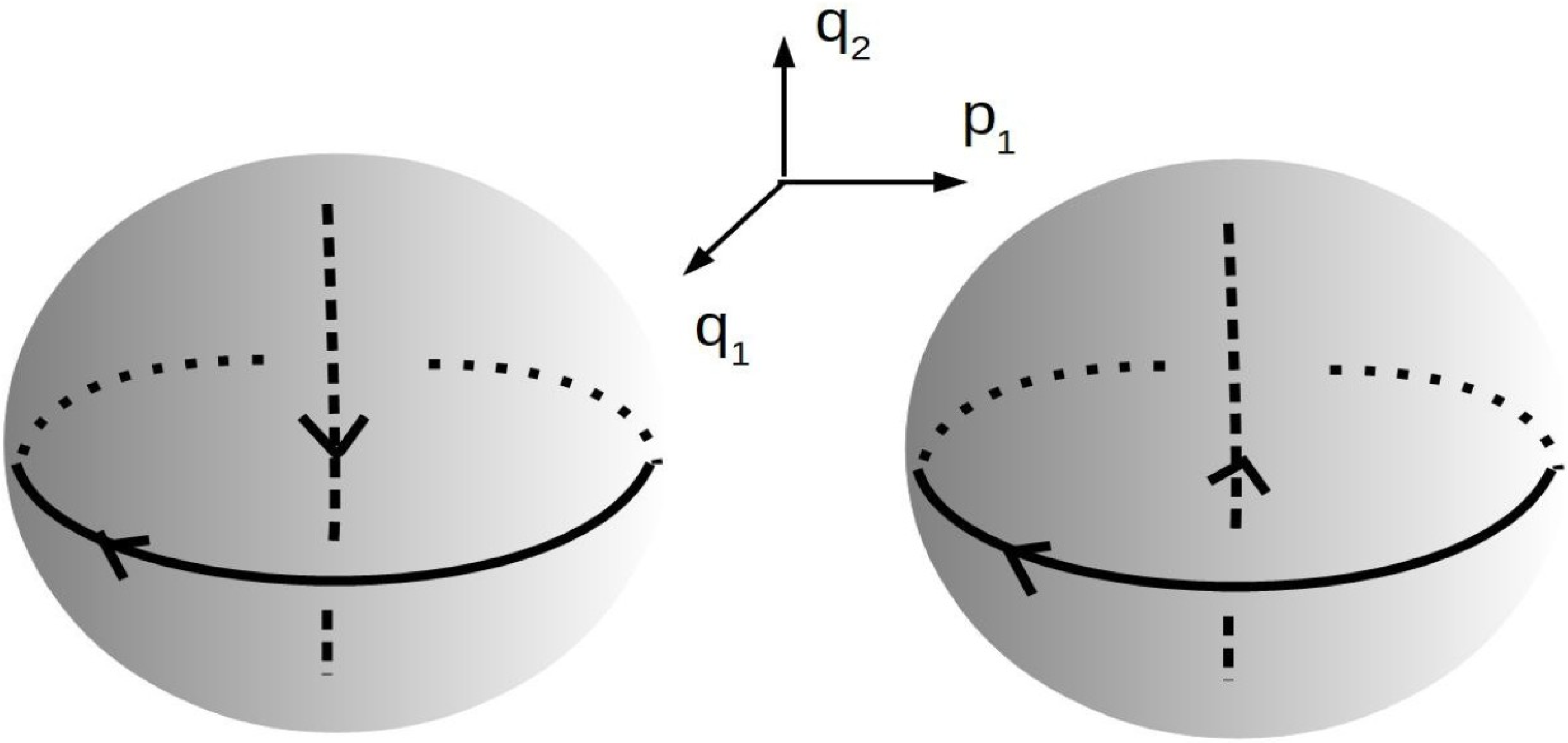
$$p_2 = -\sqrt{\left(1 - q_1^2 - p_1^2 - q_2^2\right)}$$

$$p_2 = +\sqrt{\left(1 - q_1^2 - p_1^2 - q_2^2\right)}$$


q2
p1
q1



q_2
p_1
q_1



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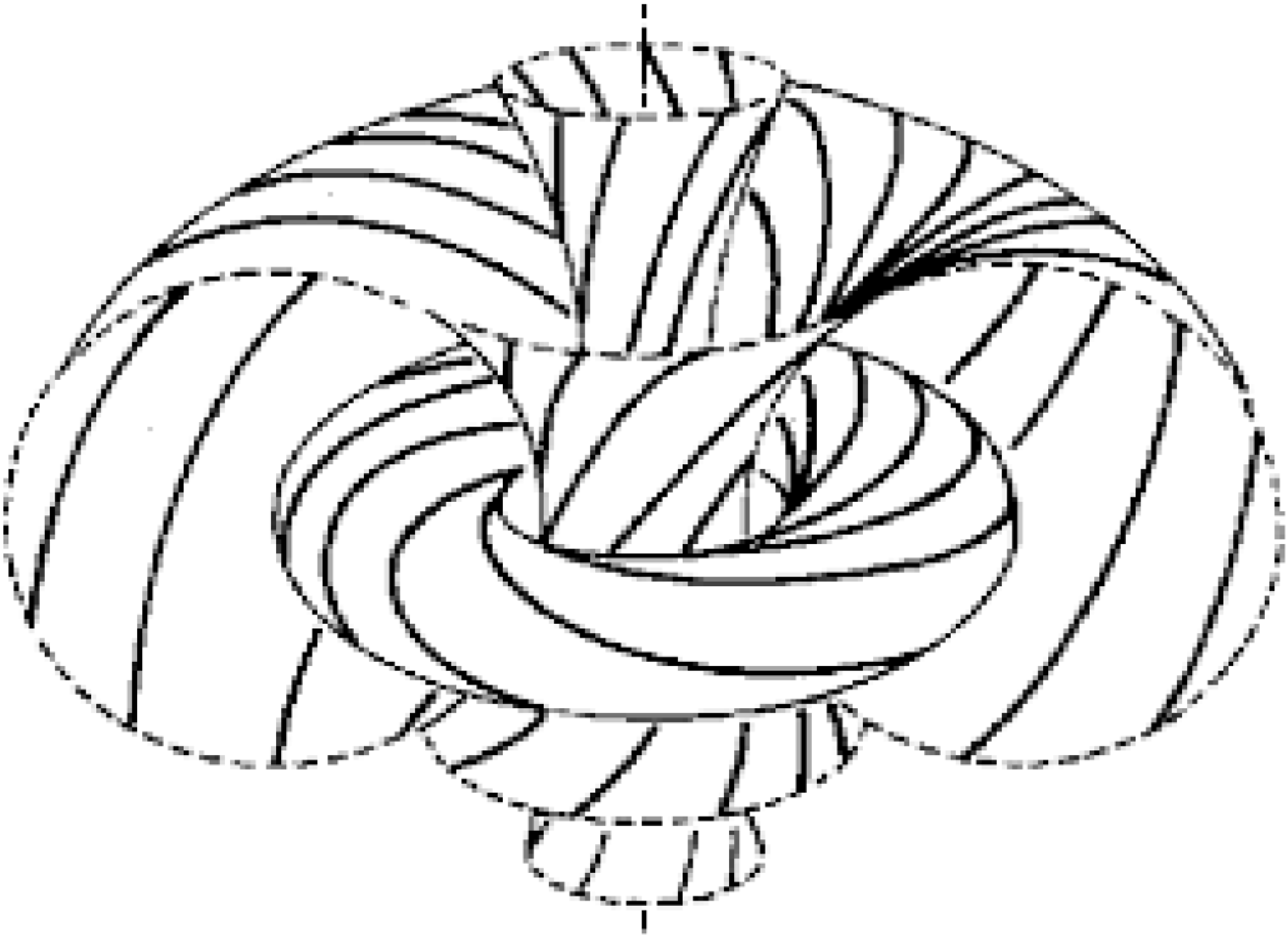
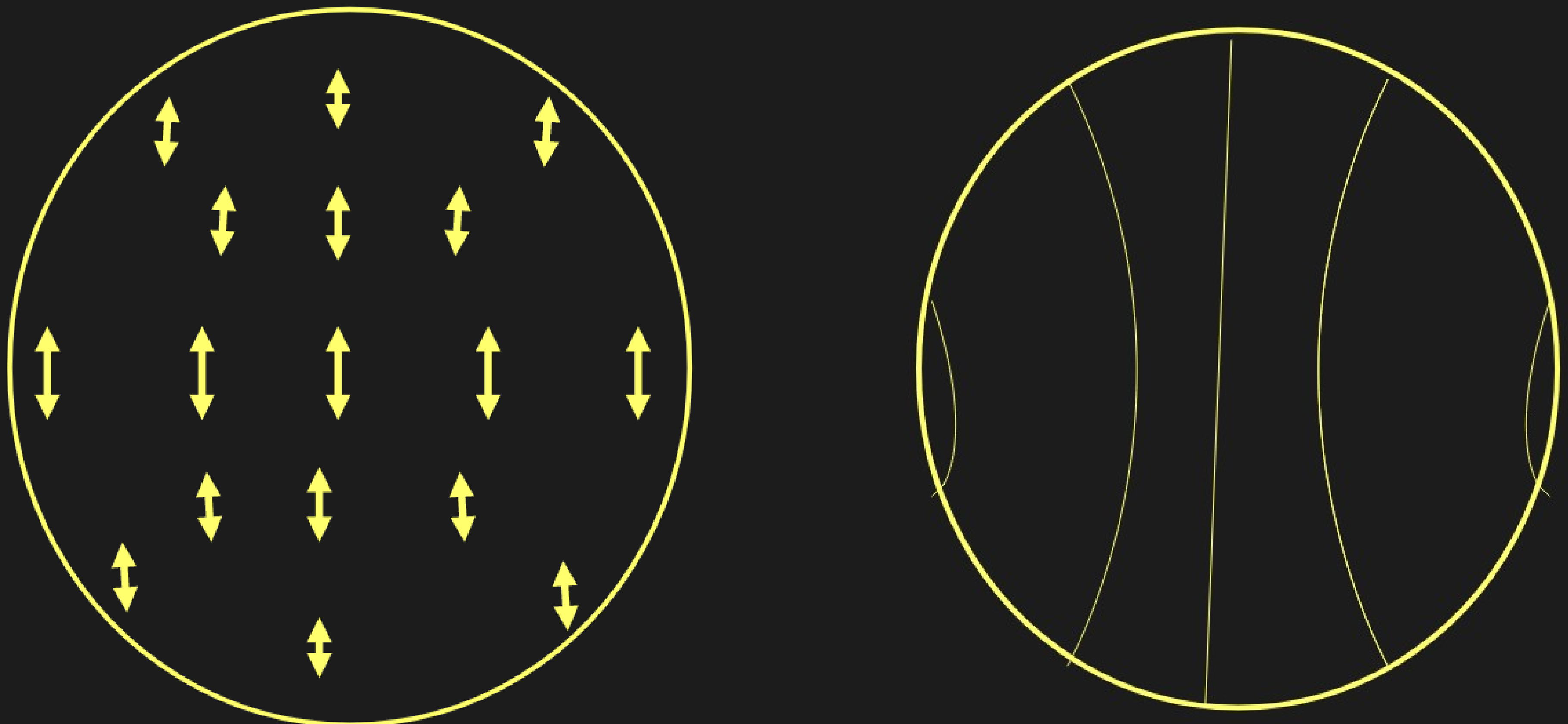
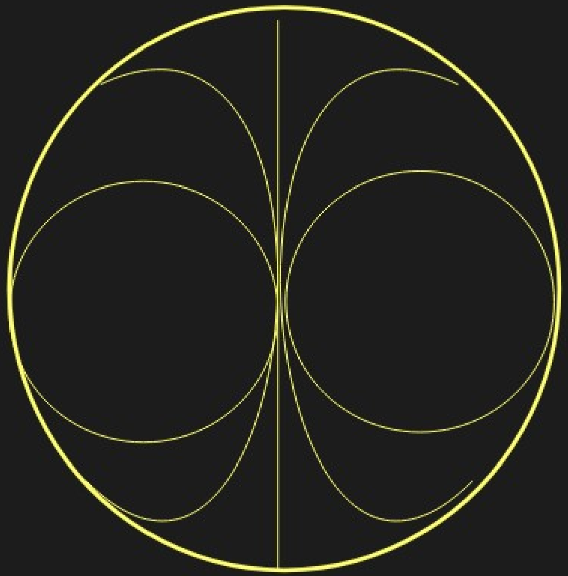


Hopf fibration in stereographic projection- from Penrose and Rindler, “Spinors and Spacetime” volume 1

Linear polarisation over the whole celestial sphere



The other side of the sphere



- Ends up with an undefined direction at the south pole
- Same as the pattern of an electric plus a magnetic dipole superposed.
- Same as the singular case of the Poincaré sphere