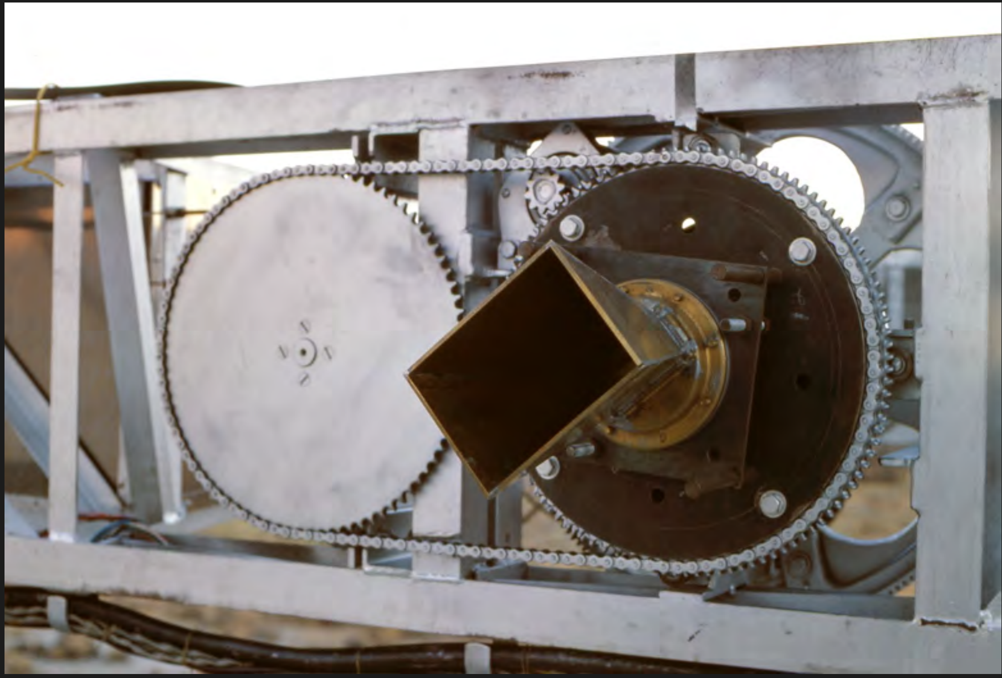
Polarisation 3

Early days of interferometry with polarisation
OVRO 1958-



ON THE MEASUREMENT OF POLARIZATION DISTRIBUTIONS OVER RADIO SOURCES

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Received August 12, 1963

ABSTRACT

A method for the determination of polarization distributions over radio sources by means of interferometric techniques is described. An expression is given for the response of an interferometer with arbitrarily polarized antennas, and some special cases of interest are discussed.

From the history of OVRO by Marshall Cohen

Synchrotron radiation is polarized, and a powerful new method for measuring polarized brightness distributions was worked out by postdocs Dave Morris, V. "Rad" Radhakrishnan, and George Seielstad (PhD '63, later OVRO assistant director), who used it to estimate the energetics and

The ‘black box’ equation of 1964

By using the definitions for I, Q, U, and V given following equation (7), the response of a two-element interferometer of the correlating type can be shown to be

$$\begin{aligned} R(t) = \tfrac{1}{2}k\{ & I[\cos \phi_1 - \phi_2) \cos (\theta_1 - \theta_2) + i \sin (\phi_1 - \phi_2) \sin (\theta_1 + \theta_2)] \\ & + Q[\cos (\phi_1 + \phi_2) \cos (\theta_1 + \theta_2) + i \sin (\phi_1 + \phi_2) \sin (\theta_1 - \theta_2)] \\ & + U[\sin (\phi_1 + \phi_2) \cos (\theta_1 + \theta_2) - i \cos (\phi_1 + \phi_2) \sin (\theta_1 - \theta_2)] \\ & + V[\cos (\phi_1 - \phi_2) \sin (\theta_1 + \theta_2) + i \sin (\phi_1 - \phi_2) \cos (\theta_1 - \theta_2)]\} , \end{aligned} \quad (8)$$

where $k = GA \exp [-i(2\pi s_x \Omega t + \psi)]$. The response of any particular type of interference polarimeter can be obtained by the elimination of the appropriate terms in this general expression. We may consider some special cases.

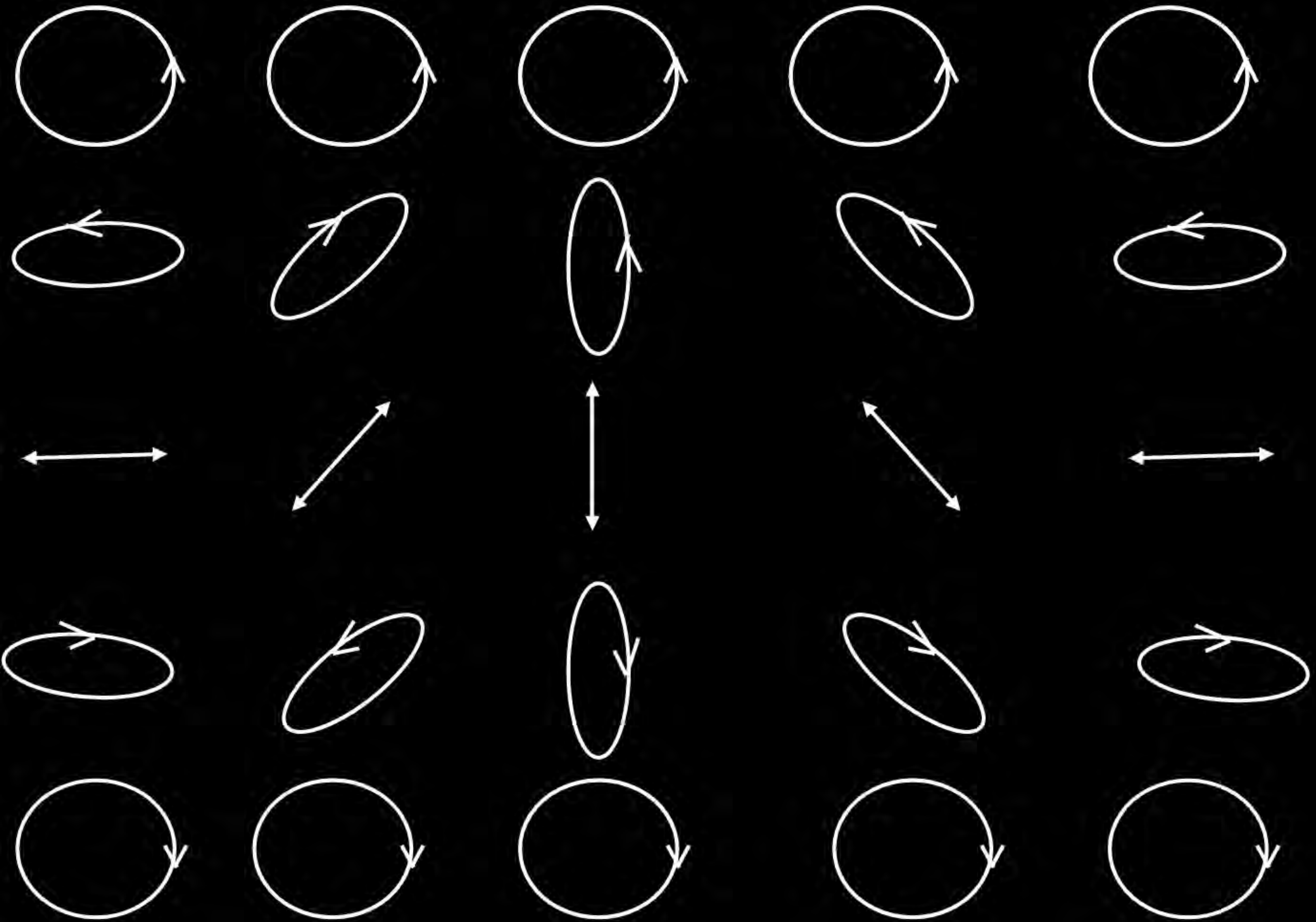

Middle author of the MRS black box equation
(Pictures courtesy W.M.Goss, J.A.Roberts)

Polarised interferometry uses conventions for phase

- We need to assign phases for the feeds – in practice they are linear or circular
- Conventionally, the linear feeds are in phase when their superposition gives linear at 45 degrees.
- Right and left circular are ‘in phase’ when their superposition gives linear along x.
- What convention for general elliptic feeds did the black box authors use?

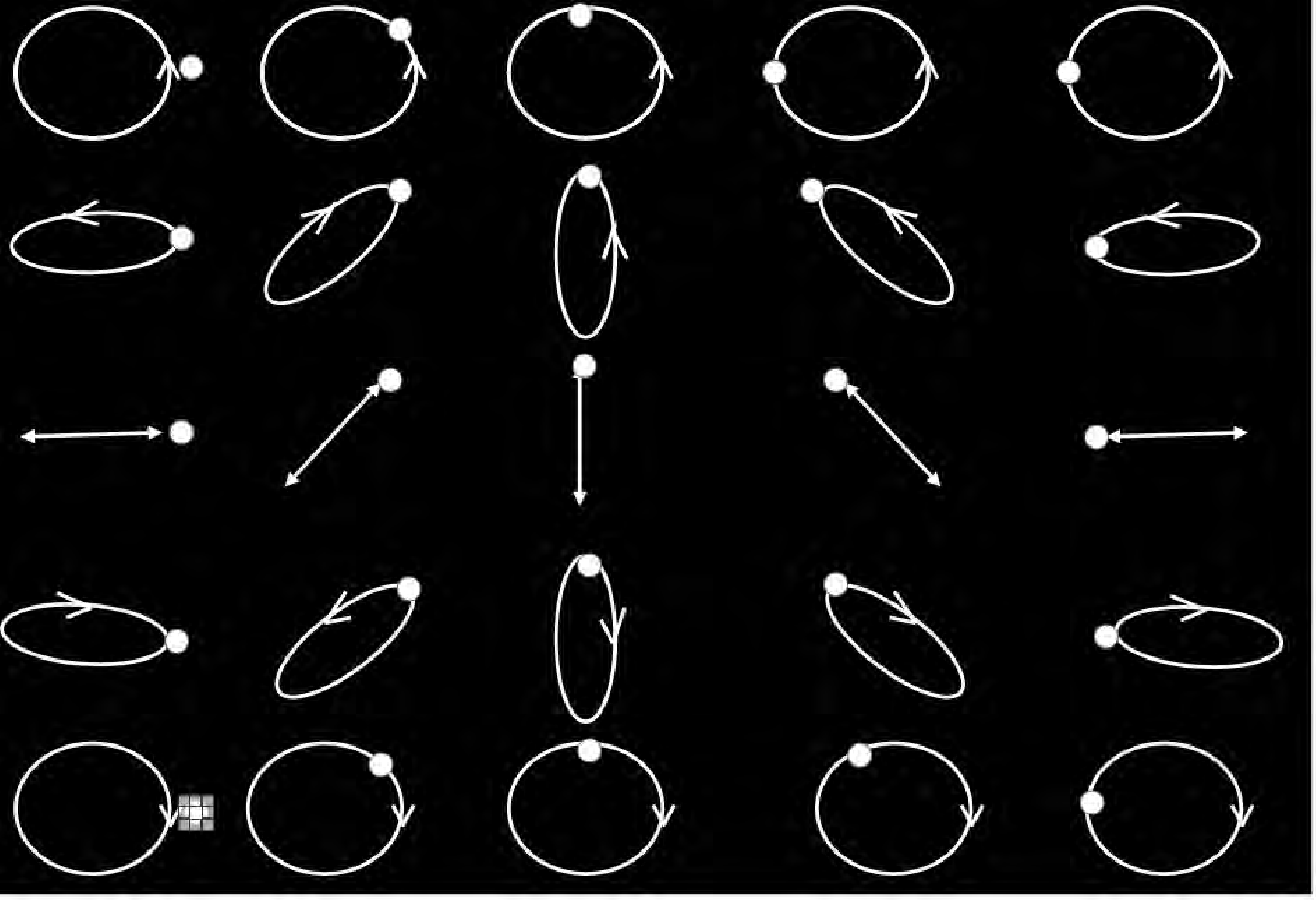
Global phase convention - a luxury?

- Radio interferometrists have got along perfectly well with their existing phase convention
- Finding – or not finding – such a global convention is still an interesting problem



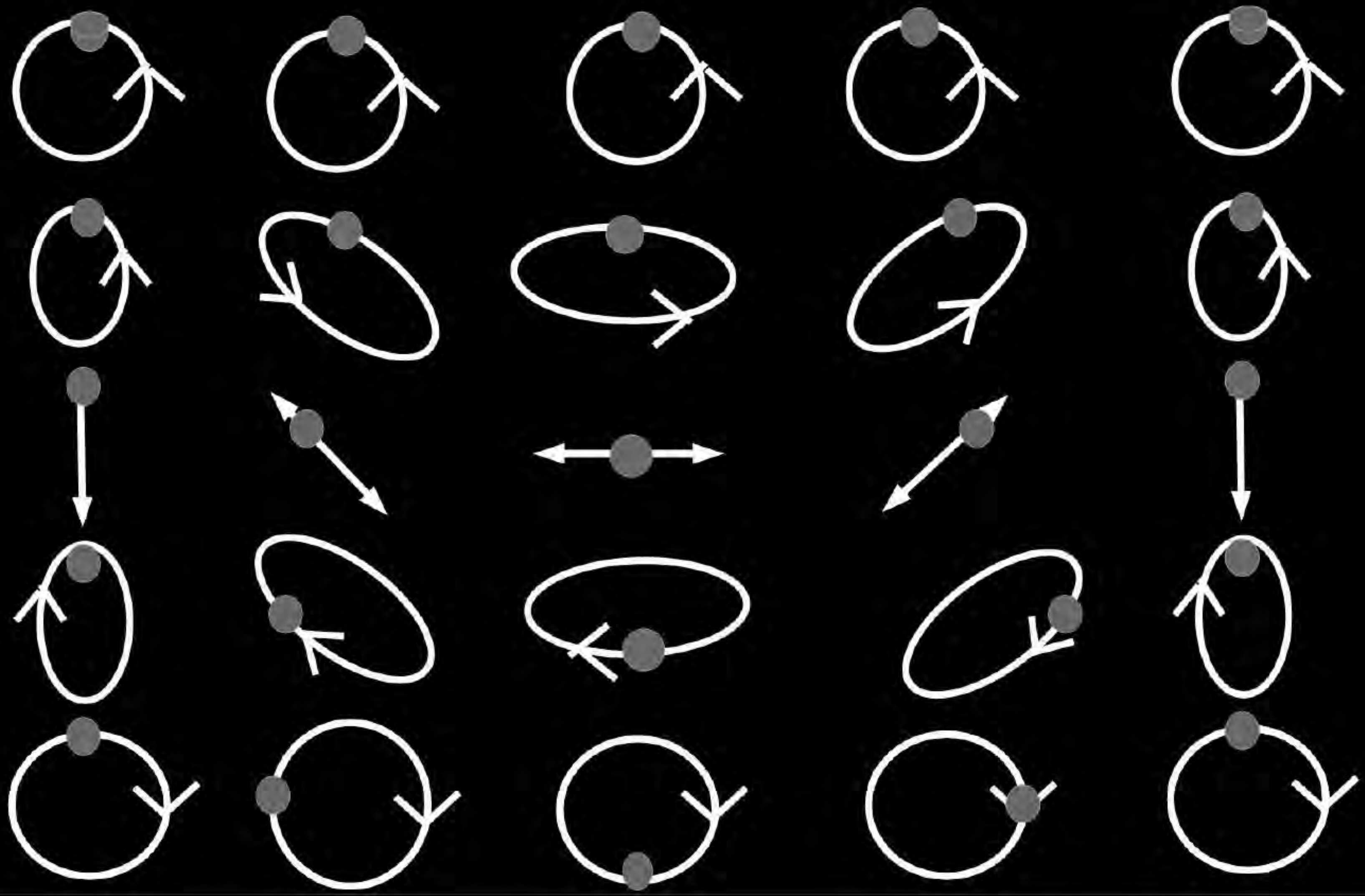
Challenge:
find a phase convention which is without any discontinuity.

Put a dot on each of the states to mark the zero of phase in a smooth manner



We need a phase convention to define Stokes visibility between differently polarised antennas

“Major axis” is ill defined at poles and along the IDL



This convention does better We choose phase of any state to be in phase with right circular.

Works ***almost*** everywhere

Two is company

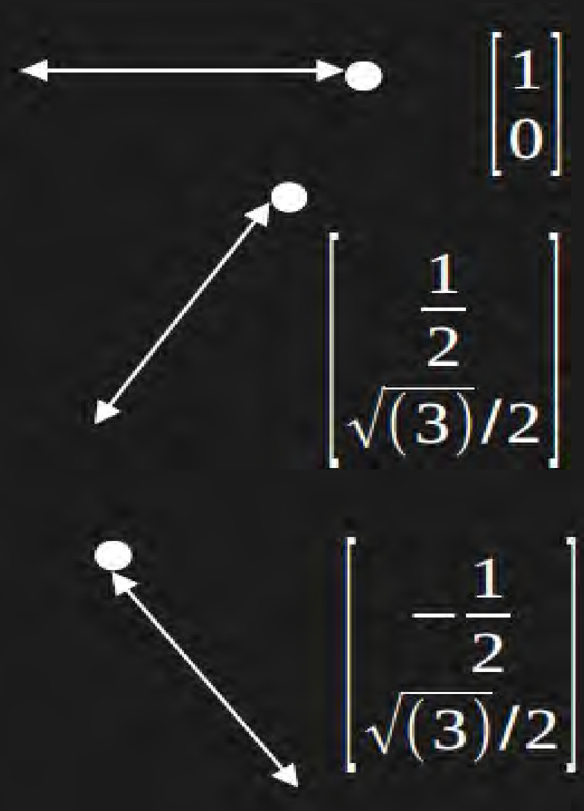
- We have a formula for the interference term, ,
  explored for orthogonal feeds, and closure phase of non ideal feeds $E_a^{+} E_b$
- With just two *non orthogonal* beams, one can always adjust the relative phase to make $E_a^{+} E_b$ real, so that we get maximum intensity
- This physically reasonable definition of phase difference was adopted by Pancharatnam (1956) in his studies of the interference of polarised light

Extract from 'Generalised Theory of Interference', Proc. Ind. Acad. Sci **44**, 247 (1956) by S. Pancharatnam.

the value of δ as defined above. Hence we will be guilty of no internal inconsistency if we make the following statement by way of a definition: *the phase advance of one polarised beam over another* (not necessarily in the same state of polarisation) *is the amount by which its phase must be retarded relative to the second, in order that the intensity resulting from their mutual interference may be a maximum.*

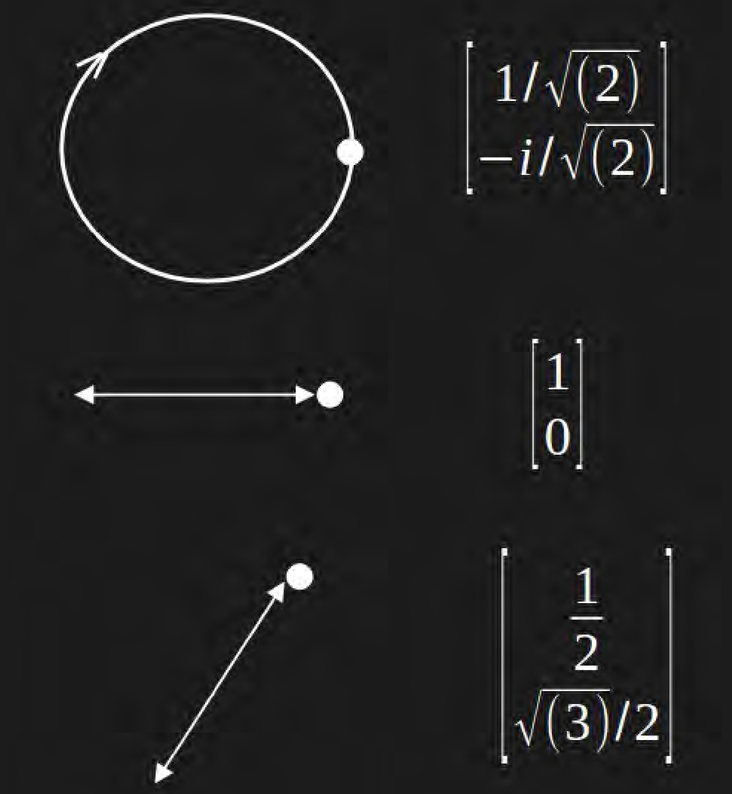
This phase advance is identically equal to δ, and the above definition holds only for non-orthogonal vibrations.

Three is a crowd

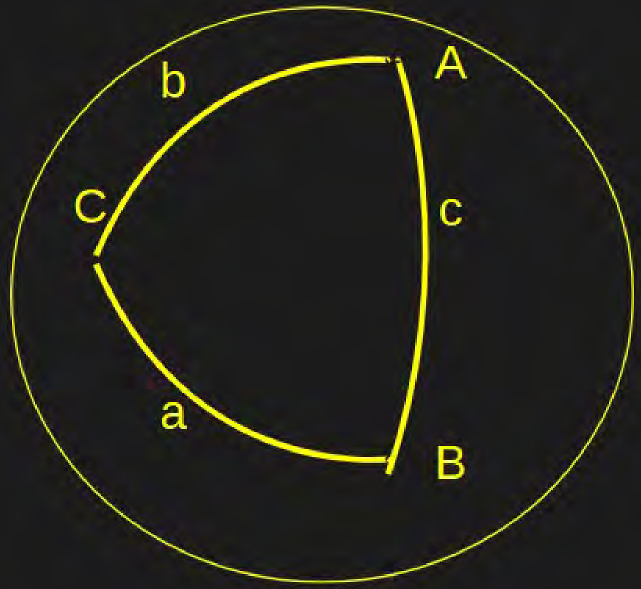


"In phase with " satisfies two of the criteria of an "equivalence relation" A~A and A~B implies B~A

but not the third. It is not "transitive", A ~B and B~C But C ~A need not be true



One slide course   on spherical trigonometry



- The ‘sides’ *a,b,c* are geodesics / great circles
- Angles at any vertex are are no problem
- Sum is greater than π
- Excess is area!

“An unexpected geometrical result”

V. When a beam of polarisation C is decomposed into two beams in the states of polarisation A and B respectively, the phase difference δ between these beams is given by

$$|\delta| = \pi - \tfrac{1}{2}|E'| \qquad (5\,a)$$

where the angle $|E'|$ is numerically equal to the area of the triangle C′BA colunar to ABC. (E′ is also the spherical excess of the triangle C′BA, *i.e.*, the excess of the sum of its three angles over π.)

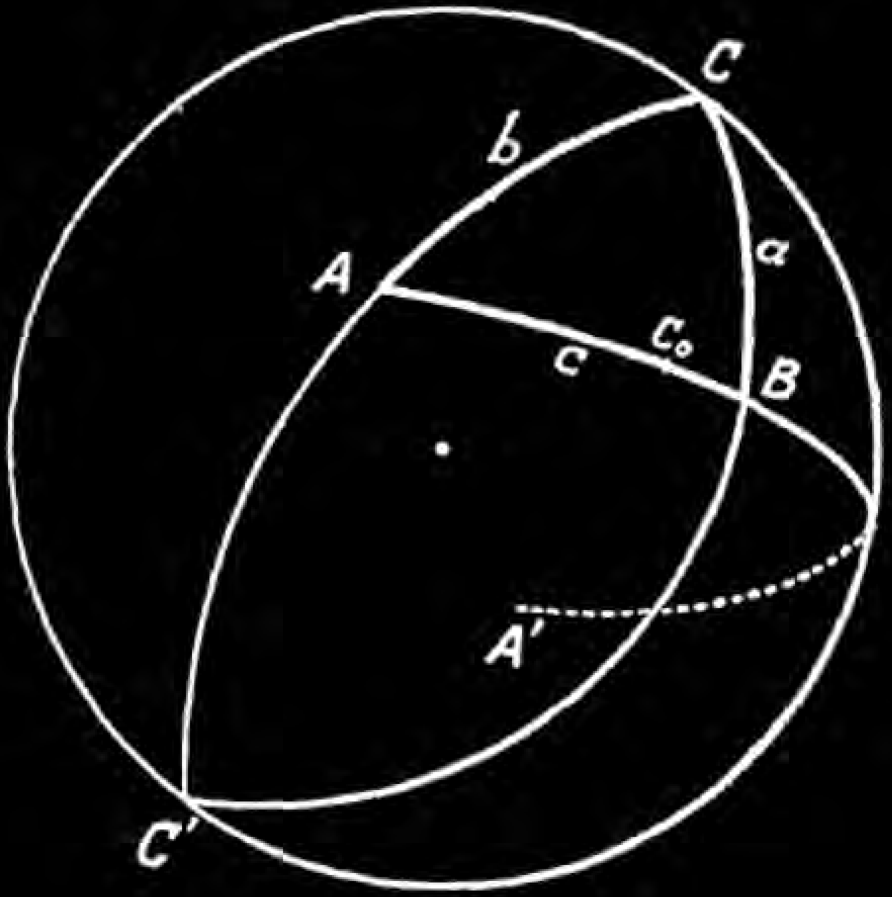


FIG 2

Geometry of the basic result of the paper.

The only property used throughout the paper is the magnitude $\cos^2(\gamma/2)$ of the overlap between two states

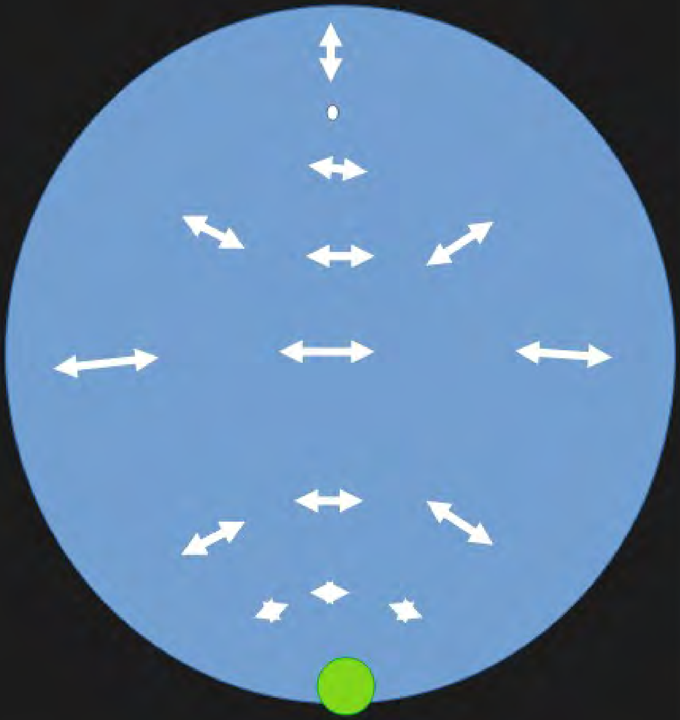
Putting phase back into the Poincare sphere

- We are still keeping the intensity fixed, so we have three degrees of freedom
- These live on a “three sphere”, as shown by the formula

$$I = z_1^* z_1 + z_2^* z_2 = p_1^2 + q_1^2 + p_2^2 + q_2^2$$

- Each point on this moves in circle, parametrised by ωt as time goes over one period
- These circles fill S^3 but it is not possible to put a dot on each of them in a continuous way! It is ‘twisted’ like a Mobius strip.(Clifford 1873, Hopf 1931) Details available on request.

Sky polarisation (Chandrasekhar, Elbert, Nature 1951)



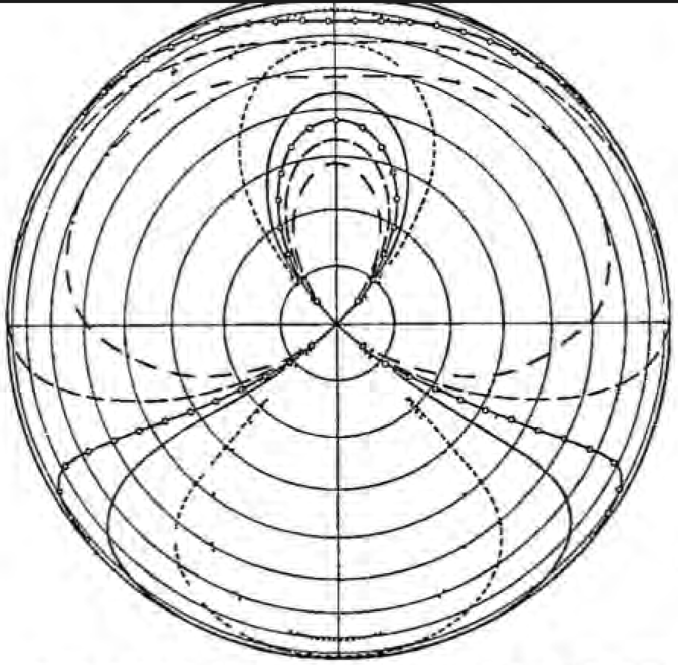


Fig 3 The neutral lines as predicted by the theory The various curves refer to the following zenith distances of the sun

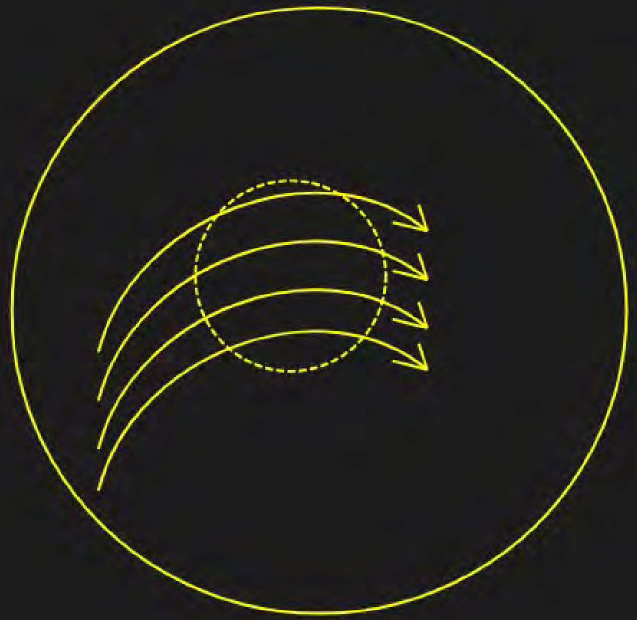
$\theta_0 = 90°$, ——— $\theta_0 = 58\ 7°$, — — $\theta_0 = 43\ 9°$,
- - - - - $\theta_0 = 76\ 1°$, -o-o-o- $\theta_0 = 50\ 2°$, — — $\theta_0 = 36\ 9°$,

The curves in this figure which roughly correspond to Dorno's observations are marked similarly

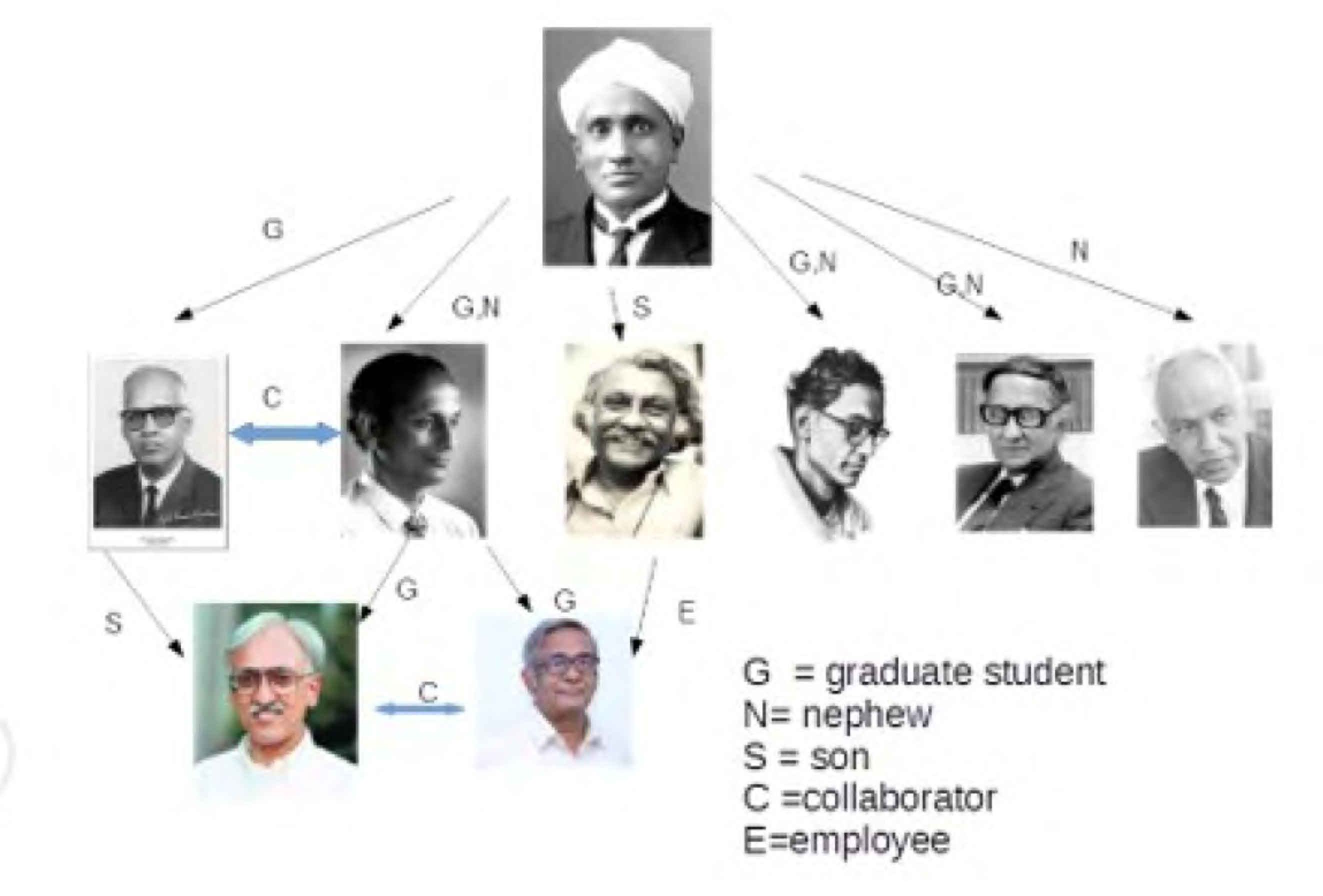
Going off axis

- If we make a polarised map of the whole sky in Stokes parameters (e.g CMB) or we want to characterise the polarisation of an antenna over the full solid angle, we need a convention for polarisation as a function of direction.
- Linear doesnt work – “hairy ball theorem”. We can move the problem to a single point (Kildal)
- There is a solution which does work by bringing in elliptic polarisation – details on request

There are two places where there is no wind



- “index” is the signed number of turns a vector makes with respect to your direction of travel around a closed curve. The sketch shows index -1 for counterclockwise traversal of the loop
- Index can only change when the loop passes a point where the vector field vanishes
- We can make the index change by +2



A ‘family tree’ of people who have all worked on polarisation, and are related academically, genetically, or both.

Most relevant to our theme is the work of S.Pancharatnam, fourth from left in the second row.

Radhakrishnan, co-author of the black box formula, is third from left