Polarisation 2

- Unpolarised light is that which fails every test for polarisation
- E_x , E_y cannot be single frequency and still explain unpolarised light
- However, ‘monochromatic’ is a fiction
- Early 19 century models had a single frequency with jumps of (relative) amplitude / phase between *x* and *y* to explain partial polarisation
- It needed Stokes (1851) to see the light and show it to us

The Stokes insight

- Focus on what an experimenter does
- Most general experiment (at that time) was creating some combination of the two field components with some amplitudes and phases and measuring the average intensity

$$I = \langle (a\ E_x + b\ E_y)(a\ E_x + b\ E_y)^* \rangle$$

- What is this averaging?

Quasi-monochromatic light/signals

- Consider a wave having frequencies in a small interval $\Delta\nu$ around a central frequency ν

  with the Fourier components having random phases

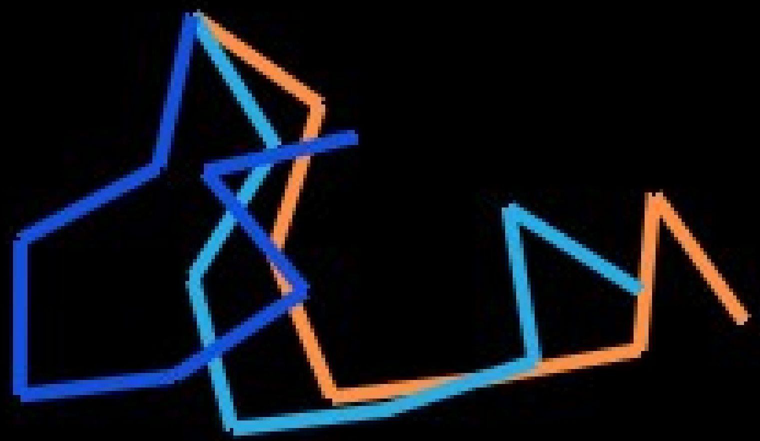
- Intuitively, for times short compared to $1/\ \Delta\nu$, the combined amplitude has a random but stable value and we are back to the monochromatic case
- For $t \sim> 1/\ \Delta\nu$ both the amplitude and phase of the combination can be quite different from $t=0$
- Stokes envisages averaging over many such ‘coherence times’ (in flowing 19th century prose)

Removing the centre frequency

$$\sum_{v_0-\frac{1}{2}\Delta v}^{v_0+\frac{1}{2}\Delta v} \exp\left(i\phi(v)-i2\pi v t\right)= ¿¿$$

$$\exp\left(-i2\pi v_0 t\right)\sum_{-\frac{1}{2}\Delta v}^{+\frac{1}{2}\Delta v} \exp\left(i\phi(v)-i2\pi\left(v-v_0\right)t\right)$$

Doing it with phasors



- Seven phasors added with frequencies ranging from – $\Delta\nu/2$ to $\Delta\nu/2$ around the central frequency
- $\Delta\nu$ t = 0, 1/8, 1/3 cycle
- Correlation falls to zero at $\Delta\nu$ t ~1 cycle

All we need is.....

$$aa^*\langle E_x E_x^*\rangle + ab^*\langle E_x E_y^*\rangle + ... = I$$

A neat separation of properties of the light, captured in four real parameters, and properties of the apparatus used to measure it

Which was ex.actly what was introduced into QM in 1927 by von Neumann, Landau

$$\text{Trace}\left(\begin{bmatrix} aa^* & ba^* \\ ab^* & bb^* \end{bmatrix}\begin{bmatrix} \langle E_x E_x^*\rangle & \langle E_x E_y^*\rangle \\ \langle E_y E_x^*\rangle & \langle E_y E_y^*\rangle \end{bmatrix}\right)$$

expressions. The combination of several independent polarized streams is next considered, and with respect to this subject a proposition is proved which may be regarded as the capital theorem of the paper. It is as follows.

When any number of independent polarized streams, of given refrangibility, are mixed together, the nature of the mixture is completely determined by the values of four constants, which are certain functions of the intensities of the streams, and of the azimuths and eccentricities of the ellipses by which they are respectively characterized; so that any two groups of polarized streams which furnish the same values for each of these four constants are optically equivalent.

It is a simple consequence of this theorem, that any group of polarized streams is equivalent to a stream of common light combined with a stream of elliptically polarized light from a different source. The intensities of these two streams, as well as the azimuth and eccentricity of the ellipse which characterizes the latter, are determined by certain formulæ, which will be found in their place.

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The Stokes parameters

¿ ¿

$$\boldsymbol{C} := \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} = \frac{1}{2} \begin{bmatrix} I+Q & U-iV \\ U+iV & I-Q \end{bmatrix}$$ ¿

More ‘anticipatory plagiarism’ he is expanding von Neumann’s density matrix in terms of Pauli matrices

The four parameters *I,Q,U,V* have a clear physical and experimental meaning – linear polarisations along 0 /90 degrees (Q positive / negative), 45 degrees / 135 degrees (U positive /negative), and circular polarisation (V positive / negative). I is the total intensity

Perfectly polarised and perfectly unpolarised radiation

- For perfectly polarised, drop the averaging sign – the complex amplitudes z_1 and z_2 are stable for ever (actually << 1/ 1/ Δν)
- The "coherency matrix" then reads $C=\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \begin{bmatrix} z_1^* & z_2^* \end{bmatrix}$ and has only three real parameters
- In the unpolarised case, the two components of the electric field have the same power but are uncorrelated, so we only get a multiple of the unit matrix, in any basis

Stokes' decomposition 1

- "Mixture of two opposite elliptically polarised beams"
- Break up the light into two incoherent, orthogonally polarised beams of (in general) unequal intensities
- Today, we would use the following decomposition of a Hermitean matrix

$$\begin{bmatrix} e_1^a \\ e_2^a \end{bmatrix} \lambda_a \begin{bmatrix} e_1^a* & e_2^a* \end{bmatrix} + \begin{bmatrix} e_1^b \\ e_2^b \end{bmatrix} \lambda_b \begin{bmatrix} e_1^b* & e_2^b* \end{bmatrix}$$

Stokes decomposition 2

Rearrange the previous expression to read

$$\boldsymbol{C}=\lambda_a\left(\begin{bmatrix} e_1^a \\ e_2^a \end{bmatrix}\begin{bmatrix} e^a{}_1*, \text{¿} e_2^a* \text{¿} \end{bmatrix} + \begin{bmatrix} e_1^b \\ e_2^b \end{bmatrix}\begin{bmatrix} e_1^b*, & e_2^b* \end{bmatrix}\right)+\text{¿¿}$$

$$(\lambda_b-\lambda_a)\left(\begin{bmatrix} e_1^b \\ e_2^b \end{bmatrix}\begin{bmatrix} e_1^b*, & e_2^b* \end{bmatrix}\right)=\text{¿¿} \quad \lambda_a I_2+(\lambda_b-\lambda_a)\left(\begin{bmatrix} e_1^b \\ e_2^b \end{bmatrix}\begin{bmatrix} e_1^b*, & e_2^b* \end{bmatrix}\right)$$

This is the incoherent sum of unpolarised light with elliptically polarised light

Stokes works in any basis !

- What happens to Q,U,V if we change basis?

- In a circular basis, we have $\begin{bmatrix} \langle E_R E_R^* \rangle & \langle E_R E_L^* \rangle \\ \langle E_L E_R^* \rangle & \langle E_L E_L^* \rangle \end{bmatrix}$
- We can now write

$$\boldsymbol{C} = \begin{bmatrix} I+V & Q-iU \\ Q+iU & I-V \end{bmatrix}$$

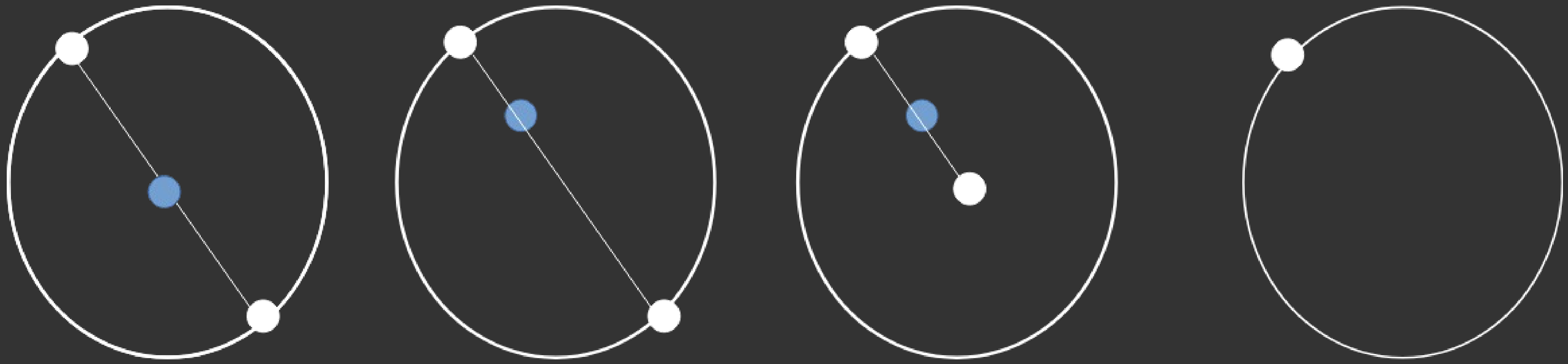
- We want $I\ Q\ U\ V$ to have their original meanings

Stokes has his own solid sphere

- two eigenvalues λ_a λ_b are intensities, so their product, the determinant, is >=0
- We therefore have $I^2 - Q^2 - U^2 - V^2 \geq 0$
- The zero determinant case is when we have perfectly polarised light since one of the eigenvalues is zero
- The “Stokes sphere” has radius *I* ,unpolarised at centre
- Partially polarised radiation lives inside it and perfectly polarised radiation lives on the surface.
- The surface is nothing but the Poincare sphere if we take the radus to be 1, i.e use Q/I, U/I, V/I as coordinates

Geometry of ***incoherent*** addition

- One can take any two points in or on the sphere, join them by a straight line, and divide according to the proportions in which one wants to mix the two beams.



Unpolarised point source, correlation between imperfect feeds

$$C = \langle (aE_f + b\,E_{\bar{f}})(cE_f + d\,E_{\bar{f}})^* \rangle = \frac{I}{2}(ac^* + bd^*)_s$$

$$¿\begin{bmatrix} c^* & d^* \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \boldsymbol{F}_B^+ \boldsymbol{F}_A$$

Some unexplained closure phases at GMRT led Sanjay Bhatnagar and me to look at the phase of the triple product of visibilities on a point source which should have been zero. What, then, is the phase of

$$(\boldsymbol{F}_A^+ \boldsymbol{F}_C)(\boldsymbol{F}_C^+ \boldsymbol{F}_B)(\boldsymbol{F}_B^+ \boldsymbol{F}_A)$$

The phase of a triple product....

- The phase of the triple product is not zero but only dependent on the state of polarisation of the feeds
- SB, RN motivated by closure errors due to polarisation leakage (2001)
- This fact was independently encoutered by Pancharatnam (1956) (crystal optics), Bargmann QM theorem (1964), Berry (QM) (1984)

- Details to follow